

## Numerical methods in European geotechnical education

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**ABSTRACT:** Numerical methods in engineering have recently achieved great development following computer evolution. However, advanced numerical modelling techniques in geomechanics have not been used at a satisfactory level (in practice). There are many reasons for this, but most of them are related to the inadequate preparation of students, researchers and practising engineers in the use of advanced numerical techniques. In this work, the authors first discuss the importance of numerical methods using recent literature. A minimum curriculum requirement for the teaching numerical methods in geomechanics is then proposed. It can be seen that the teaching of numerical methods in geomechanics is related mostly to second-cycle degree programmes.

### INTRODUCTION

Most of the engineering problems (or even real systems in general) involve complex physical phenomena [1]. To understand such phenomena, engineers need to make some simplifying assumptions, which allow them to formulate a mathematical model [2][3]. Quite frequently, the model consists of a set of Partial Differential Equations (PDE) for which (in most cases) there is no analytical solution [4][5]. In these cases, it is necessary either to simplify the model further or to obtain an approximate solution [1][2].

Numerical Analysis (NA) has undergone a spectacular development over recent decades [1] following the rapid advances and developments in modern computer technology [4][6][7]. A major part of NA involves the development and application of Numerical Methods (NM), procedures, algorithms and other numerical techniques for delivering approximate solutions for complex scientific and engineering problems. Many engineers make use of numerical modelling considering also the approximations, the assumptions and the uncertainties that are implicit in that modelling [3]. Over recent decades, the application of NM for the solution of complex engineering problems has been more firmly established in engineering practice [5]. Increasingly, more computationally-based studies are being carried out for the solution of real-life problems than ever before. Their role in verification of the results obtained from conventional analyses, especially in the cases where the validity of the latter is uncertain, is now being well recognised [8].

Finite Difference Methods (FDM) and Finite Element Method (FEM) are the most widely employed numerical techniques for the solution of differential equations [2][5][9]. Additional methods include the Boundary Element Method (BEM), the Discrete (Distinct) Element Method (DEM), the method of characteristics, as well as iterative methods referred to as prediction-correction, which nonetheless are less popular in structural mechanics. FDM were almost exclusively used for numerical solutions of geotechnical problems prior to the establishment of FEM. Lately, the interest in this method has been brought to light as it has important advantages for the simulation of specific problems. That is, problems where the variable of time is important for their solution as, for example, the flow and solidification or distribution of seismic waves [10]. FEM is considered one of the most important developments in civil engineering (CE) of the twentieth century [3][11] even if the concept dates back for several centuries [4]. Although it was developed and applied for the static and dynamic analysis of structures [4], it is actually used in a wider category of problems as, for example, in fluid mechanics, thermodynamics, acoustics and seismology [2-5][12]. The method was initially developed to solve problems considering linear material behaviour (elastic) [12], but now more complicated models can be elaborated [4][13]. Limitations on the usage of FEM concern computational time, as well as validation, verification and proximity of results [3][11][14].

Soil complexity has led engineers to resort to empirical procedures for geotechnical design [15-17]. The (unfortunate) predominance of empirically-based methods in geotechnical engineering can be explained in terms of historical

background and current design/construction practices [18]. Summarising the above mentioned statements, geotechnical engineering is still viewed mainly as empirical science. Nevertheless, the stress-strain soil response and the behaviour of structures considering soil-structure interaction through numerical, computer-oriented techniques are becoming more and more popular in soil and rock engineering [5][12]. It is becoming increasingly necessary to use numerical analysis in the design of foundations to model accurately the structure's response to loading [19]. The advances in numerical methods include solution of geometrically complex problems even in the presence of highly nonlinear material behaviour and of media, consisting of two or more phases [2][5][20] with large uncertainties in the stress-strain response and the spatial variability description of geomaterials, other complicated models [2][5][21] or even multi-scaled models [5]. Advanced theories and latest developments in geomechanics (non-linear models, gradient theories, dynamic soil-structure behaviour, coupling models) are already implemented in numerical software (mostly in FEM) [22-24]. Focus is also set on precision and stability of the algorithms, as well as computation time [25] and on the implementation of probabilistic tools in FEM [26].

The importance of NM in large or mega sized projects (underground works, off-shore works or other critical infrastructure) is well established since the corresponding geotechnical problems are solved mostly using computational methods [10][27]. In medium sized projects (except of the use on validity and verification of conventional analysis), NM can help in better understanding of the performance of structures during construction and throughout its service life [28]. But even in small sized geotechnical projects such as a building foundation, the use of numerical methods is sometimes necessary, since beam-foundations using the Winkler soil model can be solved using methods other than NM only in specific cases [29]. FEM can be also applied in simple problems (i.e. where only shear stress appears or flow problems) where elasticity cannot describe all the observed phenomena (dilation, change of pore pressures) [30].

However, the use of advanced numerical modelling techniques in geomechanics has not been applied at a satisfactory level (in practice). Felice states that NM are still viewed as an *unnecessary expense for software that will see minimal use and require expensive training with a limited return on a practices' profitability* following previous considerations [31]. In this vein, Arulmoli notices that

*Practicing geotechnical engineers are faced with challenges from different fronts including limited budgets and demanding schedules; clients, owners, and reviewers who are not well-informed about the benefits of advanced computer programs or who are unwilling to accept tools or methodologies that have not been proven in the industry [22].*

Other reasons that existed a few years ago but no longer exist include the non-user-friendly environment of available software [32] and scientific doubts concerning NM from pure mathematicians [33]. Many authors claim that most of the reasons that have caused the lack of use of NM in practice are related to the inadequate preparation of students, researchers and practising engineers in order to use such advanced tools [13][18][28][31][32]. Often tutors exhibit the lack of tools for proper teaching (i.e. Web-based lectures [34]). Finally, some educators support an extension of civil engineering curricula with management courses [35].

In this work, training of numerical methods is looked at in the first and second cycle degree programmes of higher education in CE. In the previous paragraphs, the importance of the numerical methods in civil (structural) engineering and, more specifically, in geotechnical engineering was discussed. In the next paragraphs, the implementation of NM in European CE schools is examined. Research (carried out by the authors) on the curricula of the European schools that participate in EUCEET III<sup>1</sup>, concerning NM, is presented. Then, a minimum curriculum requirement is discussed for every study cycle. Finally, some conclusions are drawn and a proposition is made for the implementation of NM in education and subsequently in practice.

## RESEARCH DESCRIPTION

The authors' research covers seventy-two educational institutes (EI) of the one hundred five educational institutes participating in EUCEET III. The resources, on which the research was based, were the information provided by every civil engineering (CE) institute on its Web page. Every course related to numerical methods was distributed according to the cycle that was offered and a categorisation that was created by the authors. For this work, the authors created three categories of course: a) introductory courses in numerical analysis (IC) and numerical methods (NM); b) courses involving NM in structural engineering in general (SC); and c) courses involving NM in geotechnical engineering (GC). It must be stressed that the boundaries of the above mentioned separation of courses is not discrete, since some courses can be placed in more than one category. The curricula in which the first and second cycles are not discretely separated (five year programmes), the first four years are regarded as the first cycle and the last year stands for the second cycle. It must be also acknowledged that research is limited to four languages (English, French, German and Greek). Results are presented in pie charts.

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<sup>1</sup> (European Civil Engineering Education and Training) with participation of 101 Civil Engineering Faculties from European Universities, as well as 15 European and National Professional Organisations, coordinated by the Ecole National de Ponts et Chaussées in Paris and supported by the European Commission.

## RESULTS AND COMMENTS

Figure 1 includes the distribution of courses in study cycles for every category defined previously (IC, SC, GC) and generally for NM courses. The first general conclusion that can be drawn from Figure 1(a) is that almost all CE institutes have at least one course involving NM or NA in general.

This is a significant observation that supports the importance of NM in any level of CE studies. The minority of 4% consists of institutes in which CE curriculum is extended in management courses. In the same figure, it can be noticed that 10% of the institutes do not include any lesson in the first cycle of studies. These are institutes that still consider that NM and NA in general, cannot be considered as a fundamental part of CE studies. They regard it as special knowledge that must be taught in the second cycle study programmes.

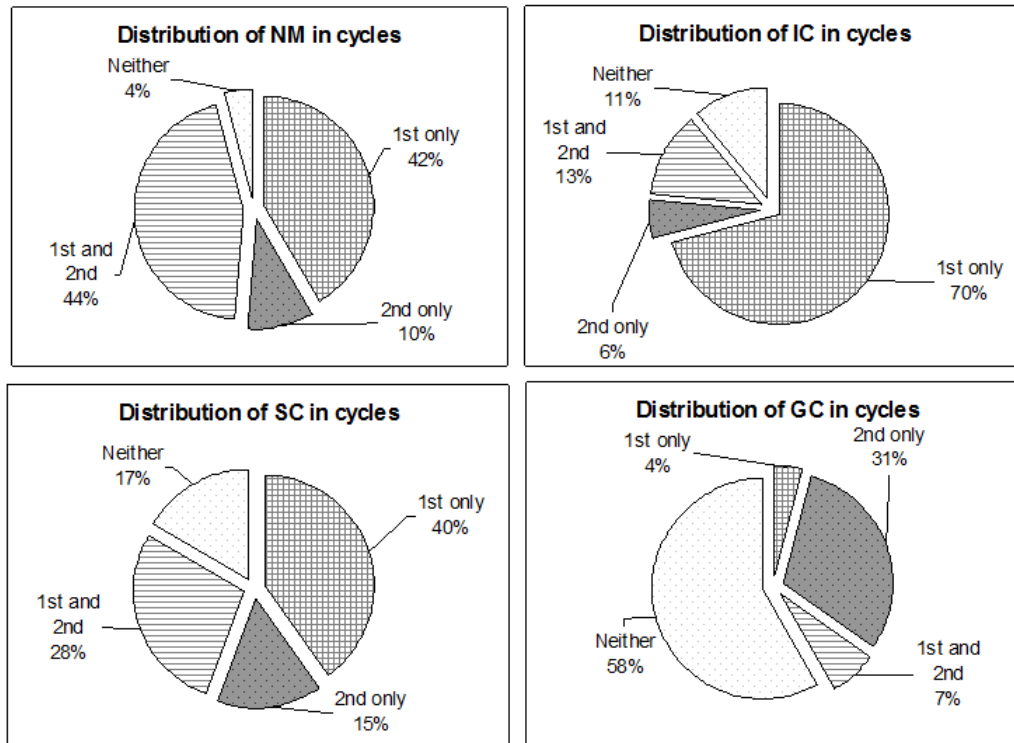


Figure 1: Pie charts showing the percentage of EI teaching (a) any kind of course, (b) introductory courses, (c) structural courses, (d) geotechnical courses concerning NM or NA in the first cycle.

A more detailed analysis of the courses is presented in Figure 1(b), Figure 1(c) and Figure 1(d), following the categorisation of the courses. As it can be seen in Figure 1(b), the majority of EI has, at least, one IC in NM (89%) and most of them have, at least, one IC in the first cycle of studies (83%). This is an expected result since IC should be offered in fundamental studies.

The 7% of EI that have NM courses but not IC is involving curricula that IC are condensed and combined mostly with SC. The 7% of EI that include IC only in the second cycle of studies is part of the 10% observed in Figure 1(a). The remaining 3% in Figure 1(a) consists of EI that have only SC or/and GC in the second cycle of studies.

The results of more specific NM courses in the first cycle are shown in Figure 1(c) and Figure 1(d). As shown in Figure 1(c), the majority of EI accepts the importance of teaching SC in general (83%) and mostly (68%) in the first cycle programmes. The 13% of EI that have NM courses but not SC is involving curricula that SC are condensed and combined with IC or GC. It can be noticed that the 40% have SC only in the first cycle.

This part of EI assumes that SC is a fundamental part of CE studies. However, knowledge of SC can be also enhanced in the second cycle programmes (43%). In contrast, only 11% of CE departments in EI consider NM in geotechnical engineering as a fundamental course. The comparison of Figure 1(c) and Figure 1(d) shows the difference of importance for the EI of NM in SC and GC: in the first cycle programmes a difference of 57% is noted.

As far as GC are concerned, the pie chart in Figure 1(d) supports the observation that specialised knowledge regarding NM and geotechnical engineering is offered mostly in the second cycle degree programmes. European EI consider GC to be specialised courses and, since the basic knowledge (IC and most of SC) is included in the first cycle programmes, most of GC can be taught in the second cycle degree programmes.

## Minimum Curriculum Requirement

The theoretical background for most computer methods that are examined in courses is extensive and it cannot be covered in the frames of only one course. For this reason, more undergraduate and postgraduate courses are needed for the detailed description of various computational methods for a minimum level of the education needed for the use of NM [10]. It is also accepted that effective teaching of numerical methods combines the above mentioned extended theoretical part with computer-based applications and presentations [32][34]. These statements support the notion that more undergraduate and postgraduate courses are needed for the detailed description of computational methods in CE studies.

Reyer states that *...Numerical methods are fundamental concepts in engineering* [36]. From the point a view of practising engineers, Felice states that the *new project delivery systems will make the knowledge of (the use of) NM in geotechnical engineering, necessary, in all sizes of projects* [31]. These statements can be linked with a major part of the first part (Introduction) of this work to support the statement of necessity of GC in the fundamental part of studies.

Based on the above considerations, some criteria concerning minimum curriculum requirement are posed in this work. At least one course in every category of courses for the first cycle of studies (assuming that GC is fundamental) sets a minimum criterion [Criterion 1 (At least three NM courses in the first cycle – one GC at least) – Figure 2]. A similar criterion can also be proposed with the assumption that GC is needed in the second cycle of studies [Criterion 2 (At least two NM courses in the first cycle – one GC at least in the second cycle) – Figure 2]. A similar criterion can be also proposed if we assume that GC is needed in the second cycle of studies [Criterion 2 (At least two NM courses in the first cycle – one GC at least in the second cycle) - Figure 2].

According to several authors [21][36], some CE departments try to compact their curricula, combining courses. Consequently, a looser criterion can be posed: three courses in NM with one GC, in any cycle, seem (even if it is a broader criterion) also similar with the minimum criterion [Criterion 3 (At least three NM courses - one GC at least) - Figure 2]. But if we consider the first paragraph of this section, then a new series of criteria can be posed. Considering that three courses are not enough, then four courses are the new minimum for teaching NM, Criterion 4 (At least four NM courses in the first cycle – one GC at least), Criterion 5 (At least three NM courses in the first cycle – one GC at least in the second cycle), Criterion 6 (At least four NM courses - one GC at least) respectively - Figure 3).

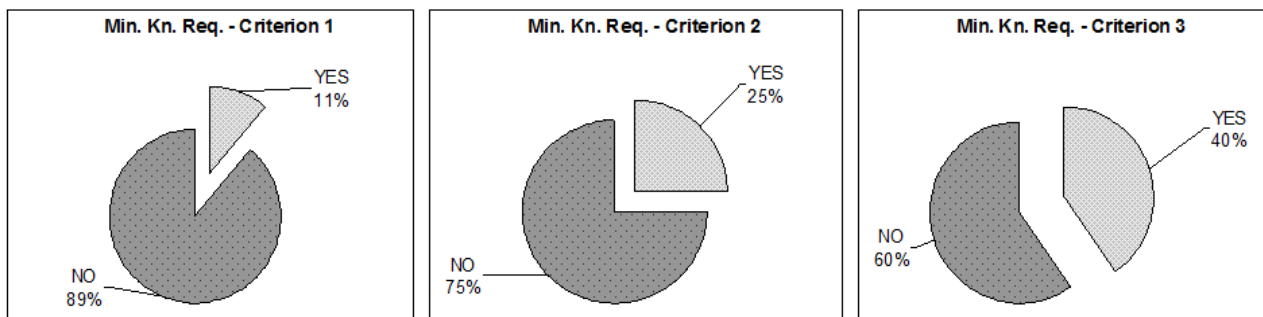


Figure 2: Pie charts showing the percentage of EI satisfying minimum curriculum criteria

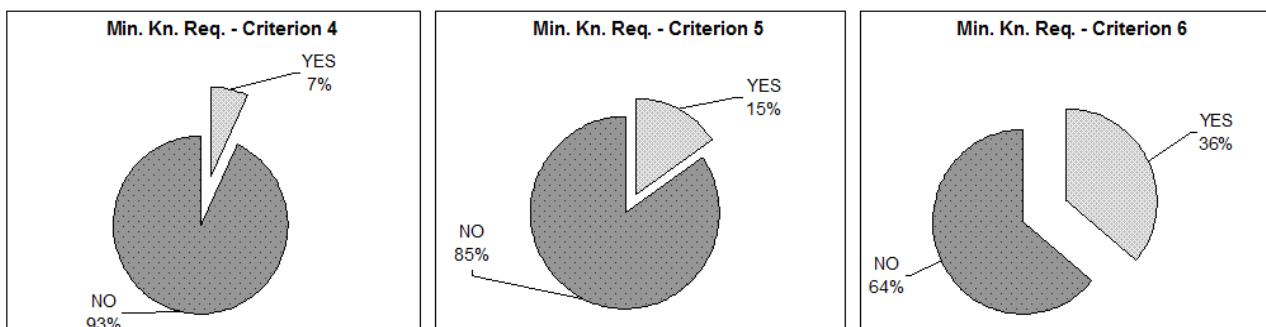


Figure 3: Pie charts showing the percentage of EI satisfying necessary curriculum criteria.

If the criteria posed above are accepted, it can be seen that the majority of the EI concerned in this research satisfies none of them. Only 7% satisfy Criterion 4 and 40% satisfy the weakest criterion (Criterion 3).

The above mentioned statements support also the general statement discussed in the introduction that the existing course curriculum in NM is insufficient for the implementation of NM in practice. Nevertheless, existing curricula

can form the basis for further implementation of NM in education and, subsequently, in practice of geotechnical engineering.

## CONCLUSIONS – PROPOSALS

The statement ...*The implementation of modern advanced geotechnical modelling (from research to practice) passes through education and training...* is something that is well stated among specialists of all areas [18][22][31][33]. Recently, that is in 2005, a workshop took place in Maryland, USA, where the main topic was the implementation of non-linear modelling of geotechnical problems in practice. Desai proposes better interaction between practice and theory, in particular, between construction, design and research [37]. This proposal is fundamental for the broadness of the NM in general. Hueckel suggests that encouragement of the use of numerical analysis applying FEM in undergraduate geotechnical engineering classes, should stimulate the use of mathematical modelling in the graduate geotechnical (not only geomechanical) classes as a part of required (core) course education, find the ways that major industry and research stimulate the development of new and use of existing advanced mathematical models [38]. This opinion lies in the area of the suggestions made by Desai [37] in a more detailed form. Muir Wood proposes implementation of recent developments in geomechanics in the first cycle programmes, using simple terms [28]. As mentioned earlier, a practicing engineer Felice states that NM concerning geotechnical engineering will be necessary in the near future and focus must be placed on the broadness and the conditions for the use of advanced numerical techniques [31]. All the above mentioned assertions verify the statements mentioned earlier that ...*Numerical methods are fundamental concepts in engineering* [36].

As far as this work is concerned, it can be summarised as follows:

- Numerical Methods are a fundamental part of civil engineering studies in European academic institutions, in general;
- European study programmes include mostly introductory and structural courses in the fundamental studies and mostly structural courses in the second cycle of studies;
- Geotechnical courses are mostly related to the second cycle study programmes.
- Minimum curriculum requirements as addressed in this work are not met for NM courses and, even more, if GC are considered.

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